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Some Defuzzification Methods for Interval Type-2 Pentagonal Fuzzy Numbers

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Abstract

Interval type-2 pentagonal fuzzy numbers are developed from the pentagonal fuzzy numbers and interval-valued pentagonal fuzzy numbers concepts. Previous researchers have suggested that various defuzzification methods were used to transform pentagonal fuzzy numbers into crisp numbers. However, very little research discusses defuzzification methods for interval type-2 pentagonal fuzzy numbers. Five interval-tuple fuzzy numbers that act as the input transformed information are needed to obtain crisp numbers via defuzzification methods. Therefore, this study examined some defuzzification methods for developing interval type-2 pentagonal fuzzy numbers where interval type-2 pentagonal fuzzy numbers (input) are transformed into crisp numbers (output). In addition, a comparison between interval type-2 pentagonal fuzzy numbers and general pentagonal fuzzy numbers are provided to validate the consistency and efficiency of these defuzzification methods.

Keywords: interval type-2 fuzzy numbers; pentagonal fuzzy number; defuzzification method.

1 Introduction

In the real world, imprecision is inevitable because of unexpected situations. Fuzziness is frequently used to describe imprecision and uncertainty. Zadeh [38] developed the fuzzy sets concept to deal with the uncertainty of information in real-life problems. Chang and Zadeh [4] identified the fundamental fuzzy sets concept and associated it with numbers, where a fuzzy number is a tool used to represent a vague inflexible notion numerically. To deal with different problems involving uncertainty, imprecision and vagueness, the fuzzy sets concept was extended to various other concepts, especially interval-valued fuzzy sets by Sambuc [31], intuitionistic fuzzy sets by Attanassov [1] and interval type-2 fuzzy sets by Mendel et al. [20]. Type-2 fuzzy sets proposed by Zadeh [39] as an extension the idea of an ordinary fuzzy set, or a type-1 fuzzy set. The number of α-planes that approximate the generalized type-2 fuzzy sets is reduced with the implementation of high-order α-planes integration [23]. Due to the computational difficulty of applying a generic type-2 fuzzy set, Liang and Mendel [17] defined an interval type-2 fuzzy set as a specific instance of a mathematical formulation for the type-2 fuzzy set concept when the secondary membership degree equals one. A comparison between interval type-2 fuzzy systems and general type-2 fuzzy systems is described by Ontiveros et al. [24] for a set of diagnosis problems. In short, different types of fuzzy sets are defined to clarify their vagueness and uncertainty. The fuzzy numbers concept is an extension of the real numbers concept. In previous studies [9], there were several types of fuzzy numbers that were developed with the membership function concept, including triangular, trapezoidal and pentagonal fuzzy numbers.

In the case of ordinal numbers, a fuzzy number has values that are ambiguous and imprecise [35]. A triangular fuzzy number [2] or trapezoidal fuzzy number [11] is a commonly used fuzzy number to represent various decision-makers' opinions. To deal with real-world problems that usually involve more than four tuples, pentagonal fuzzy numbers were introduced by Ponnivalavan and Pathinathan [29] with certain operational properties. Pathinathan and Ponnivalavan [27] also proposed the general notion of pentagonal fuzzy numbers using set theoretic techniques. The concept of a fuzzy centre in a pentagonal fuzzy number was introduced by Rajkumar and Pathinathan [16]. Johari et al. [15] expand the relationship between the sums of squares and the sums of centered pentagonal numbers induced by partitions of eight. Pathinathan and Ajay Minj [26] introduced the interval-valued pentagonal fuzzy number and developed a relationship between the pentagonal intuitionistic fuzzy number and the interval-valued pentagonal fuzzy number. Evolving from the generalization of pentagonal fuzzy numbers and interval-valued pentagonal fuzzy numbers, Ajay Minj and Pathinathan [21] proposed interval type-2 pentagonal fuzzy numbers and discussed three types of regular interval type-2 pentagonal fuzzy numbers. These numbers become more complex and comprehensive with interval characters. Therefore, it is better to transform interval type-2 pentagonal fuzzy numbers to a single number as a product of the fuzzy model using the defuzzification method.

The defuzzification method is referred to as the reverse fuzzification process, where fuzzy numbers are transformed into crisp numbers. There are numerous defuzzification methods that are described in the literature, including the centre of area [6], bisector of area [7], largest of maxima [18], smallest of maxima [8], and mean of maxima [13], just to name a few. The defuzzification process was considered in fuzzy numbers, where Chen and Chen [5] presented a procedure for sorting trapezoidal fuzzy numbers according to their centre of gravity. Following type-reduction and defuzzification, Wahab et al. [36] discussed the data points of the type-2 fuzzy on-uniform rational B-spline curve. Shankar [33] developed a centroid method for trapezoidal fuzzy numbers and applied the float time for each activity in the fuzzy project network to identify the crucial path. The centroid method for octagonal fuzzy numbers was applied by Maheswari et al. [19] to determine the critical path and contrast it with the triangular and trapezoidal fuzzy numbers. Peddi

[28] suggested a novel approach for ranking fuzzy numbers through the defuzzification process by maximizing and minimizing sets of triangular fuzzy numbers, which were derived from extended trapezoidal fuzzy numbers. Hence, in order to reduce the overall cost of a fuzzy inventory model, several defuzzification techniques were utilized for triangular fuzzy numbers, trapezoidal fuzzy numbers, pentagonal fuzzy numbers, and hexagonal fuzzy numbers [30]. Sengupta et al. [32] used the defuzzification method for pentagonal fuzzy numbers presented using the expected value criterion to address a carbon cost integrated solid transportation problem. Three defuzzification methods, namely the centroid, mean of Alpha (α)-cut method and removal of area methods were used by Chakraborty et al. [3] to defuzzify the pentagonal fuzzy number. Pathinathan and Mike Dison [25] studied traditional defuzzification methods to organise the pentagonal fuzzy number input and produce a crisp output. However, as interval type-2 pentagonal fuzzy numbers is a direct extension of the pentagonal fuzzy numbers, its representation is still limited to the defuzzification methods. Considering this limitation, it becomes interesting to discuss some defuzzification methods where interval type-2 pentagonal fuzzy numbers as input variables are transformed into crisp numbers.

The main contribution of this paper:

i. To investigate several defuzzification methods where the input variable was interval type-2 pentagonal fuzzy numbers and the output was crisp numbers.

ii. To propose three defuzzification methods to illustrate the transformation of interval type-2 pentagonal fuzzy numbers to crisp numbers.

iii. To compare the defuzzification methods of interval type-2 pentagonal fuzzy numbers with general pentagonal fuzzy numbers to validate their consistency and efficiency.

This paper is organized as follows. Section 2 elaborates on several definitions related to the objective of this study and presents some interval type-2 pentagonal fuzzy number concepts. Section 3 discusses the defuzzification methods for interval type-2 pentagonal fuzzy numbers. Section 4 provides some numerical examples, Section 5 presents a comparative analysis, and lastly, Section 6 presents the conclusion.

2 Preliminaries

This section introduces the definitions of interval type-2 fuzzy sets and numbers, pentagonal fuzzy numbers and interval type-2 pentagonal fuzzy numbers (IT2PFN). Some concepts of fuzzy centre value, area and centroid are also presented for IT2PFNs.

2.1 Definitions of interval type-2 pentagonal fuzzy numbers

Definition 2.1. (*Type-2 fuzzy sets [20]*): *A type-2 fuzzy sets A_{T2FS} is characterized by a type-2 membership function, A type-2 fuzzy sets, $\mu_{A_{T2FS}}(x,u)$ where $x \in X$ and $u \in J_X, \subseteq [0,1]$. It is defined as:*

$$A_{T2FS} = \{((x,u), \mu_{A_{T2FS}}(x,u)) / \forall x \in X, \forall u \in J_X \subseteq [0,1]\}, \tag{1}$$

where J_x is the primary membership function in $(0, 1)$ and u is the primary membership values in $0 \leq \mu_{A_{T2FS}} \leq 1$. It can also be expressed as follows:

$$A_{T2FS} = \int_{x\in X} \int_{u\in J_X} \mu_{A_{T2FS}}(x, u)/(x, u); J_X \subseteq [0, 1], \tag{2}$$

where $\int\int$ denotes union ovel all admissible x and u.

Definition 2.2. (*Interval type-2 fuzzy sets* [20]): *An interval type-2 fuzzy set, A_{T2FS} is a type-2 fuzzy set when all secondary membership function is unity defined as:*

$$A_{T2FS} = \int_{x\in X} \int_{u\in J_X} 1/(x, u); J_X \subseteq [0, 1]. \tag{3}$$

Definition 2.3. (*Interval type-2 fuzzy numbers* [20]): *An interval type-2 fuzzy set defined on real line R is called an interval type-2 fuzzy number A_{IT2FN}. It is defined as:*

$$A_{IT2FN} = [A^U, A^L], \tag{4}$$

where A^U and A^L are upper and lower membership functions, respectively, such that $A^U \subseteq A^L$.

Definition 2.4. (*Pentagonal fuzzy numbers* [22]): *A pentagonal fuzzy number, A_{PFN} is a fuzzy subset of real line R defined as:*

$$A_{PFN} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5), \tag{5}$$

where $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5$ are real numbers with $\tilde{a}_1 \leq \tilde{a}_2 \leq \tilde{a}_3 \leq \tilde{a}_4 \leq \tilde{a}_5$. The membership function,$\mu_{A_{PFN}}$ should satisfied the condition as follows:

1. *$\mu_{A_{PFN}}(x)$ is a continuous function in the interval $[0, 1]$.*
2. *$\mu_{A_{PFN}}(x)$ is strictly increasing and continuous function on $[\tilde{a}_1, \tilde{a}_2]$ and $[\tilde{a}_2, \tilde{a}_3]$.*
3. *$\mu_{A_{PFN}}(x)$ is strictly decreasing and continuous function on $[\tilde{a}_3, \tilde{a}_4]$and $[\tilde{a}_4, \tilde{a}_5]$.*

Definition 2.5. (*Interval type-2 pentagonal fuzzy numbers* [25]): *An IT2PFN, A_{IT2PFN} is an interval type-2 fuzzy set of R. It is defined as:*

$$A_{IT2PFN} = \left[\tilde{A}^U, \tilde{A}^L\right], \tag{6}$$

where $\tilde{A}^U = (\overline{a}_1, \overline{a}_2, \overline{a}_3, \overline{a}_4, \overline{a}_5)$ and $\tilde{A}^L = (\underline{a}_1, \underline{a}_2, \underline{a}_3, \underline{a}_4, \underline{a}_5)$ are upper and lower IT2PFN, respectively, such that $\tilde{A}^U \subseteq \tilde{A}^L$.

In order to apply IT2PFN as the input variables, we propose the new fuzzy centre value, area and centroid concepts for IT2PFN expressed in Equation (6) and developed from the idea of fuzzy centre value, area and centroid of a pentagonal fuzzy number by Pathinathan and Mike Dison [25].

2.2 Fuzzy centre value of interval type-2 pentagonal fuzzy numbers [27]

Let $A_{IT2PFN} = (\overline{a}_1, \overline{a}_2, \overline{a}_3, \overline{a}_4, \overline{a}_5), (\underline{a}_1, \underline{a}_2, \underline{a}_3, \underline{a}_4, \underline{a}_5)$ be IT2PFN and $\mu_{A_{IT2PFN}}(x)$ be a membership function with c being the core known as the maximum membership value of IT2PFN. The fuzzy centre value of IT2PFN,F_{IT2PFN} is given below:

i. *For upper fuzzy centre value of IT2PFN,* $\overline{F}_{IT2PFN}$

$$\overline{F}_{IT2PFN} = \frac{\overline{a}_3}{2} + \frac{\overline{a}_1 + \overline{a}_5}{4}. \tag{7}$$

ii. *For lower of fuzzy centre value of IT2PFN,* $\underline{F}_{IT2PFN}$

$$\underline{F}_{IT2PFN} = \frac{\underline{a}_3}{2} + \frac{\underline{a}_1 + \underline{a}_5}{4}. \tag{8}$$

2.3 Area of interval type-2 pentagonal fuzzy numbers [32]

Let $A_{IT2PFN} = (\overline{a}_1, \overline{a}_2, \overline{a}_3, \overline{a}_4, \overline{a}_5), (\underline{a}_1, \underline{a}_2, \underline{a}_3, \underline{a}_4, \underline{a}_5)$ *be IT2PFN and the area of IT2PFN,* Ar_{IT2PFN} *is calculated as follows:*

i. *For upper area of IT2PFN,* $\overline{Ar}_{IT2PFN}$

$$\overline{Ar}_{IT2PFN} = \frac{(\overline{a}_5 - \overline{a}_1) + 2(\overline{a}_4 - \overline{a}_2)}{4}. \tag{9}$$

ii. *For lower area of IT2PFN,* $\underline{Ar}_{IT2PFN}$

$$\underline{Ar}_{IT2PFN} = \frac{(\underline{a}_5 - \underline{a}_1) + 2(\underline{a}_4 - \underline{a}_2)}{4}. \tag{10}$$

2.4 Centroid of interval type-2 pentagonal fuzzy numbers [16]

Let $A_{IT2PFN} = (\overline{a}_1, \overline{a}_2, \overline{a}_3, \overline{a}_4, \overline{a}_5), (\underline{a}_1, \underline{a}_2, \underline{a}_3, \underline{a}_4, \underline{a}_5)$ *be IT2PFN and the centroid of IT2PFN,* x_o *is defined as follows:*

i. *For upper centroid of IT2PFN,* $\overline{x}_o$

$$\overline{x}_o = \frac{\overline{a}_1 + \overline{a}_2 + 5\overline{a}_3 + \overline{a}_4 + \overline{a}_5}{9}. \tag{11}$$

ii. *For lower centroid of IT2PFN,* $\underline{x}_o$

$$\underline{x}_o = \frac{\underline{a}_1 + \underline{a}_2 + 5\underline{a}_3 + \underline{a}_4 + \underline{a}_5}{9}. \tag{12}$$

3 Defuzzification for Interval Type-2 Pentagonal Fuzzy Number

The defuzzification process plays an important role in a fuzzy environment by transforming the fuzzy number into a crisp number. The selection of a defuzzification method has a significant impact on the accuracy and stability of the model's performance [22]. A variety of defuzzification methods have been used by researchers for different types of fuzzy numbers.

This section discusses three of the defuzzification methods, namely the centre of sums, centre of gravity and weighted average methods. Also, this section proposes three defuzzification methods of IT2PFN developed from defuzzification methods of pentagonal fuzzy number by Pathinathan and Mike Dison [25]. It is believed that more effective computing methods are utilized in creating a single number in order to obtain the IT2PFN. Details of the defuzzification methods for IT2PFN are described below.

3.1 Centre of sums method [10]

The centre of sums method is the most commonly used defuzzification method, where the overlapping area is counted twice [34]. This method uses area to calculate the defuzzification value from two non-symmetric inputs.

The defuzzified value of the centre of sums method for interval type-2 pentagonal fuzzy number, x^*_{cs} is defined as follows:

$$x^*_{CS} = \frac{\sum_{i=1}^{n} F_i \cdot Ar_i}{\sum_{i=1}^{n} Ar_i} = \frac{\left(\overline{F_i} \cdot \overline{Ar_i}\right) + \left(\underline{F}_i \cdot \underline{Ar}_i\right)}{\overline{Ar}_i \cdot \underline{Ar}_i}, \tag{13}$$

where F denotes the fuzzy centre value (see Equation (7) and (8)), Ar is the area for IT2PFN (see Equations (9) and (10)) and n is the number of fuzzy sets.

3.2 Centre of gravity method [12]

The centre of gravity method is one of the common defuzzification methods that compute the area under the membership function's centre of gravity [16]. This method generates a crisp number depending on the fuzzy set's centre of gravity.

The defuzzified value of the centre of gravity method is defined for interval type-2 pentagonal fuzzy number, x^*_{CG} as follows:

$$x^*_{CG} = \frac{\int_{a_1}^{a_5} x \cdot \mu(x)\,dx}{\int_a^b \mu(x)\,dx} = \frac{\int_{a_1}^{a_2} x \cdot \mu(x)\,dx + \int_{a_2}^{a_3} x \cdot \mu(x)\,dx + \int_{a_3}^{a_4} x \cdot \mu(x)\,dx + \int_{a_4}^{a_5} x \cdot \mu(x)\,dx}{\int_{a_1}^{a_2} \mu(x)\,dx + \int_{a_2}^{a_3} \mu(x)\,dx + \int_{a_3}^{a_4} \mu(x)\,dx + \int_{a_4}^{a_5} \mu(x)\,dx}, \tag{14}$$

where x indicates the sample element and μ is the membership function.

3.3 Weighted average method [32]

The weighted average method is the most suitable method for defuzzifying two or more symmetric fuzzy inputs. This method is constructed by taking the average of each output membership function and dividing it by its respective degree of importance [25]. Additionally, it is known as the maximum membership value or the core value of a particular membership function.

The defuzzified value of the weighted average method for interval type-2 pentagonal fuzzy number, $x^*_{WA}$is defined as follows:

$$x^*_{WA} = \frac{\sum x_o \cdot \mu(x_o)}{\sum \mu(x_o)} = \frac{(\overline{x}_o \cdot \mu(x_o)) + (\underline{x}_o \cdot \mu(x_o))}{\mu(x_o)}, \tag{15}$$

where x_o represents the centroid of IT2PFN (see Equations (11) and (12)) and μ is the membership function.

4 Numerical Example

This section presents examples of transformations from IT2PFN to a crisp number using the defuzzification methods described in Section 3. It also provides an illustration of pentagonal fuzzy numbers retrieved from Pathinathan and Mike Dison [25] and transformed into IT2PFN. There are some differences according to the elements of the IT2PFN used between the example for centre of sums method compared to the example for centre of gravity method and weighted average method. This is due to the symmetric and non-symmetric form of these defuzzification methods.

4.1 Centre of sums method

Example 4.1. *Let us consider the two generalized non-symmetric IT2PFNs as follows:*

$A_{IT2PFN} = (1, 2, 3, 4, 6; 0.8), (0, 1, 2, 3, 5; 0.8)$ *and* $B_{IT2PFN} = (3, 4, 5, 6, 6.5; 0.47), (2, 3, 4, 5, 5.5; 0.47)$ *with core values of 0.8 and 0.47, respectively. Figure 1 below illustrates these two generalized non-symmetric IT2PFNs.*

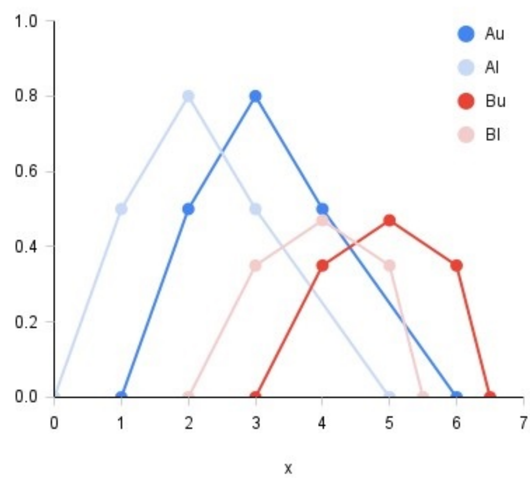

Figure 1: Non-symmetric IT2PFNs for centre of sums method.

First, the fuzzy centre value of IT2PFN is calculated using Equations (7) and (8), where $\overline{F}(A_u) = 3.25$, $\underline{F}(A_l) = 2.25$, $\overline{F}(B_u) = 4.875$ *and* $\underline{F}(B_l) = 3.875$. *Based on Equations (9) and (10), the area of IT2PFN is calculated as follows,* $\overline{Ar}(A_u) = 2.25$, $\underline{Ar}(A_l) = 2.25$, $\overline{Ar}(B_u) = 1.875$ *and* $\underline{Ar}(B_l) = 1.875$. *Then, the defuzzified value of two IT2PFNs was calculated using Equation (13) as follows:*

$$x* = \frac{(3.25 \times 2.25) + (2.25 \times 2.25) + (4.875 \times 1.875) + (3.875 \times 1.875)}{2.25 + 2.25 + 1.875 + 1.875} = 3.4886.$$

Figure 2 shows the defuzzified value of the two IT2PFNs.

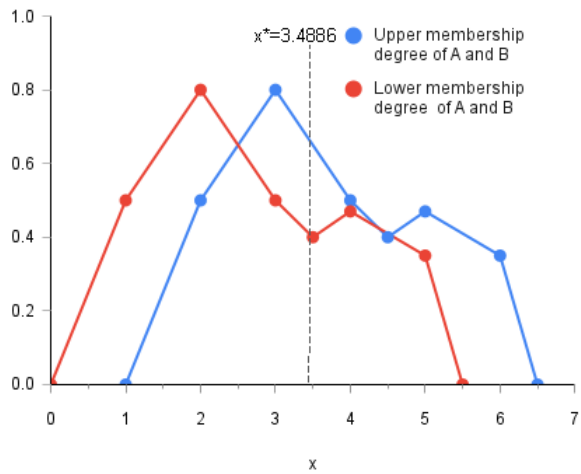

Figure 2: Defuzzified value of the centre of sums method.

According to Figure 2, the defuzzified output for upper and lower membership degree of the centre of sums method is 3.4886. This shows that the defuzzification method for centre of sums maps two generalized non-symmetric IT2PFNs into a crisp number.

4.2 Centre of gravity method

Example 4.2. *Let* $A_{IT2PFN} = (1, 2, 3, 4, 5; 0.8), (0, 1, 2, 3, 4; 0.8)$ *and* $B_{IT2PFN} = (3, 4, 5, 6, 7; 0.7), (2, 3, 4, 5, 6; 0.7)$ *be defined as two generalized symmetric IT2PFNs with core values of 0.8 and 0.7, respectively. These two generalized non-symmetric IT2PFNs are illustrated in Figure 3 below:*

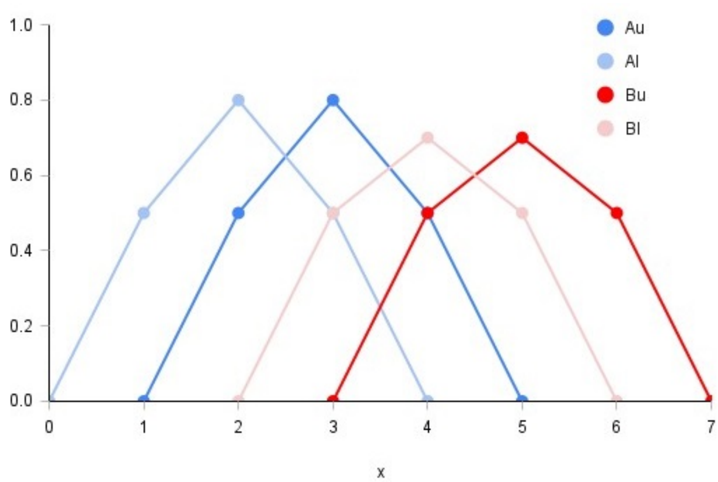

Figure 3: Two IT2PFNs for centre of gravity method.

The upper membership function is $\int_{a_1}^{a_5} x \cdot \mu(x)\, dx = 7.9$ *and* $\int_{a_1}^{a_5} \mu(x)\, dx = 2$*, while the lower mem-*

bership function is $\int_{a_1}^{a_5} x \cdot \mu(x)\,dx = 5.9$ *and* $\int_{a_1}^{a_5} \mu(x)\,dx = 2$. *The defuzzification of two IT2PFNs is calculated using Equation* (14) *as follows:*

$$x* = \frac{7.9 + 5.9}{2 + 2} = 3.45.$$

Figure 4 shows the defuzzified values of the two IT2PFNs.

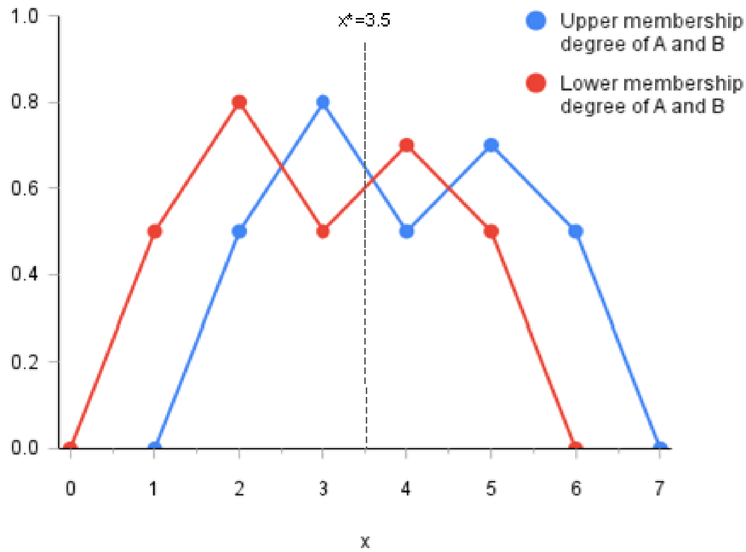

Figure 4: Defuzzified values of the centre of gravity method.

Based on Figure 4, the defuzzified results of two generalized non-symmetric IT2PFNs is 3.5 for centre of gravity. This method produced two generalized symmetric IT2PFNs as inputs to be converted into crisp outputs.

4.3 Weighted average method

Example 4.3. *Let us consider the two generalized symmetric IT2PFNs with core values of 0.8 and 0.7 as follows:* $A_{IT2PFN} = (1, 2, 3, 4, 5; 0.8), (0, 1, 2, 3, 4; 0.8)$ *and* $B_{IT2PFN} = (3, 4, 5, 6, 7; 0.7), (2, 3, 4, 5, 6; 0.7)$. *Figure 5 illustrates this two generalized symmetric IT2PFNs as follows:*

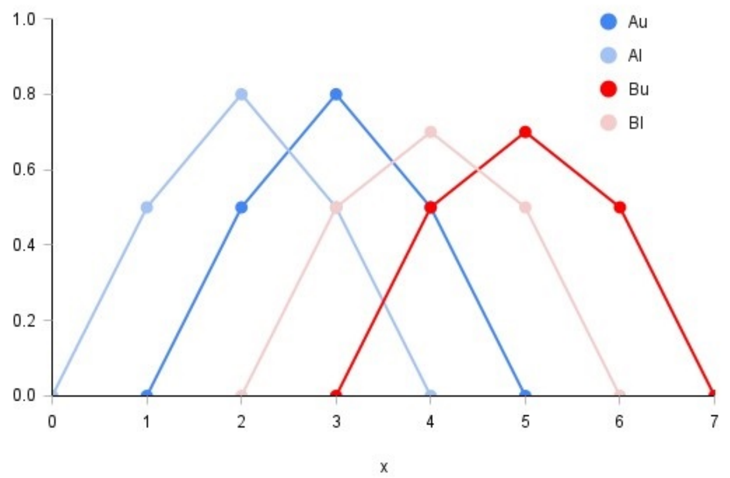

Figure 5: Two IT2PFNs for the weighted average method.

The defuzzified value of two IT2PFNs is calculated using Equation (15) as follows:

$$x* = \frac{(3 \times 0.8) + (2 \times 0.8) + (5 \times 0.7) + (4 \times 0.7)}{0.8 + 0.7 + 0.8 + 0.7} = 3.4333,$$

where the centroid of the IT2PFN derived using Equations (11) and (12) are $\overline{x}_o\left(A_u\right) = 3$, $\underline{x}_o\left(A_l\right) = 2$, $\overline{x}_o\left(B_u\right) = 5$ *and* $\underline{x}_o\left(B_l\right) = 4$*. Figure 6 shows the defuzzified values of the two IT2PFNs.*

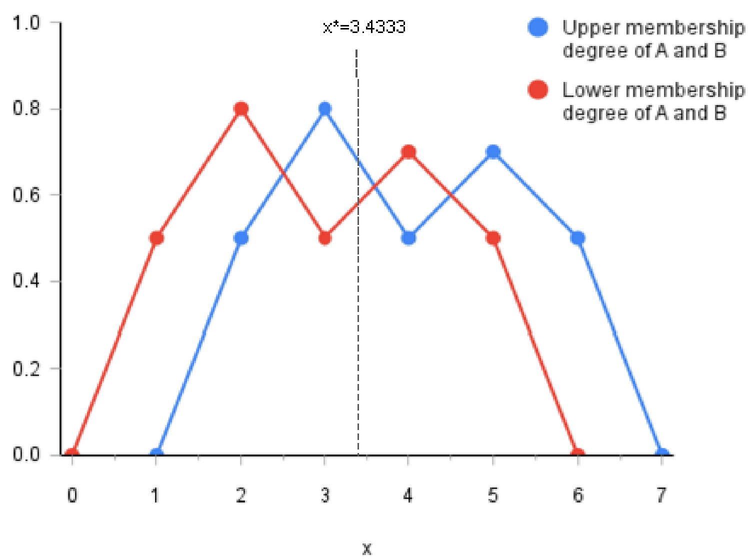

Figure 6: Defuzzified values of the weighted average method.

The crisp value of two generalized symmetric IT2PFNs using the weighted average method is 3.4333 (see Figure 6). This method provides a crisp value based on the weighted average method for a IT2PFN.

Table 1: Comparison of three defuzzification methods.

Defuzzification methods	**Defuzzified value**		
	Pentagonal fuzzy number [22]	**Intuitionistic pentagonal fuzzy number**	**Interval type-2 pentagonal fuzzy number (proposed)**
Centre of sums	3.9886	3.4375	3.4886
Centre of gravity	3.95	3.1598	3.45
Weighted average	3.9333	3.3296	3.4333

5 Comparative Analysis

In this section, three defuzzification methods, namely centre of sums, centre of gravity and weighted average, are compared based on the defuzzified value between the pentagonal fuzzy number, intuitionistic pentagonal fuzzy number and IT2PFN. Table 1 summarizes the results of defuzzification using IT2PFN with a classification of different defuzzification methods.

Table 1 shows the different results of defuzzified values when comparing the three defuzzification methods. Results above propose that the defuzzification methods for IT2PFN are suitable as they involve interval characters to reflect the uncertainty in assigning membership degrees. It also provides upper and lower membership functions to represent uncertainty and vagueness in information and fuzziness. Defuzzification methods for intuitionistic pentagonal fuzzy numbers are used for membership degrees, which are a pair of membership degrees and non-membership degrees. Each type of fuzzy number has a specific requirement for a defuzzification method to find the crisp value. Based on the results above, the defuzzified value for the centre of sums method indicates that this method possesses the property of continuity better than the centre of gravity method and the weighted average method, which is an essential requirement for IT2PFN. A minor change in the fuzzy input should not lead to a significant change in its output [14]. Hence, it can be concluded that the proposed defuzzification methods for IT2PFN are more realistic and intuitive when captured in real life, especially in relation to decision-making.

6 Conclusions

Defuzzification is not obligatory especially when identifying a single output set and the linguistic results are sufficiently informative. However, a defuzzification method is required if there are multiple fuzzy output sets for producing a single crisp value. Although fuzziness assists in the evaluation of the rules, the output of a fuzzy system must nonetheless be a crisp number. The aggregate output fuzzy set becomes the input in the defuzzification process and the result is a single number [37]. This study has proposed three defuzzification methods, namely the centre of sums, centre of gravity and the weighted average methods to determine the crisp number for IT2PFN. In order to compare the outputs using different defuzzification methods, two types of fuzzy numbers were tested. This study recommends different defuzzification methods such as first maxima method, last of maxima method, center of largest area and others to convert the IT2PFN into a single crisp number in future research. We also recommend some examples of applications will be applied in this proposed defuzification methods where IT2PFN as the input and produce crisp output.

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Conflicts of Interest The authors declare no conflict of interest.

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